Developmental Mathematics with Applications and Visualization: Prealgebra, Beginning Algebra, and Intermediate Algebra 2nd Edition
Author(s): Krieger, Terry | Rockswold, Gary

14.2 Multiplication and Division of Rational Expressions

Review of Multiplication and Division of Fractions • Multiplication of Rational Expressions • Division of Rational Expressions

A LOOK INTO MATH ▶

Stopping distance for a car can vary depending on the road conditions. If the road is slippery, more distance is needed to stop. Also, cars that are traveling downhill require additional stopping distance. Rational expressions are frequently used by highway engineers to estimate the stopping distance of a car on slippery surfaces or on hills. (See Example 4.)

In previous chapters we reviewed how to add, subtract, multiply, and divide real numbers and polynomials. In this section we show how to multiply and divide rational expressions; in the next section we discuss addition and subtraction of rational expressions.

Review of Multiplication and Division of Fractions

To multiply two fractions we use the property

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}.$$

In the next example we review multiplication of fractions. (For a full review of fractions, see Chapter 4.)

TEACHING TIP
Be sure to review how to multiply and divide fractions before showing students how to multiply and divide rational expressions.

EXAMPLE 1 **Multiplying fractions**

Work rational expressions the same way you do fractions.

Multiply and simplify your answers to lowest terms.
(a) $\frac{3}{7} \cdot \frac{4}{5}$ **(b)** $2 \cdot \frac{3}{4}$ **(c)** $\frac{4}{21} \cdot \frac{7}{8}$

TEACHING EXAMPLE 1
Multiply and simplify your answer to lowest terms.
(a) $\frac{3}{4} \cdot \frac{5}{8}$ (b) $3 \cdot \frac{5}{9}$ (c) $\frac{8}{15} \cdot \frac{3}{4}$
ANS. (a) $\frac{15}{32}$ (b) $\frac{5}{3}$ (c) $\frac{2}{5}$

c) $\frac{4}{21} \cdot \frac{7}{8}$

Cross cancel when you can.

To divide two fractions we change the division problem to a multiplication problem by using the property

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}.$$

EXAMPLE 2 **Dividing fractions**

Division is multiplication by the reciprocal.

Divide and simplify your answers to lowest terms.
(a) $\frac{1}{3} \div \frac{5}{7}$ **(b)** $\frac{4}{5} \div 8$ **(c)** $\frac{8}{9} \div \frac{10}{3}$

TEACHING EXAMPLE 2
Divide and simplify your answer to lowest terms.

a) $\frac{1}{3} \div \frac{5}{7}$

b) $\frac{4}{5} \div 8$

c) $\frac{8}{9} \div \frac{10}{3}$

Only cross cancel multiplication. Not division, addition or subtraction of fractions.

Multiplication of Rational Expressions

Multiplying rational expressions is similar to multiplying fractions.

PRODUCTS OF RATIONAL EXPRESSIONS

To multiply two rational expressions, multiply the numerators and multiply the denominators. That is,

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD},$$

where B and D are nonzero.

READING CHECK

- How do we multiply rational expressions?

EXAMPLE 3 **Multiplying rational expressions**

TEACHING EXAMPLE 3

Multiply and simplify to lowest terms. Leave your answers in factored form.

b) $\frac{5w}{3w+2} \cdot \frac{3w+2}{w-1}$

The fraction bar only cancels factors, not terms.

c) $\frac{x^2-9}{x+6} \cdot \frac{x+6}{x-3}$

d) $\frac{x^2-2x-3}{5x} \cdot \frac{10x}{x^2+5x+4}$

Now Try Exercises 29, 31, 41, 45

▶ **REAL-WORLD CONNECTION** The next example illustrates a rational expression that is used in highway design.

EXAMPLE 4 Estimating stopping distance

TEACHING EXAMPLE 4
For a similar example, do Exercise 74.

If a car is traveling at 60 miles per hour on a slippery road, then its stopping distance D in feet can be calculated by

$$D = \frac{3600}{30} \cdot \frac{1}{x},$$

where x is the coefficient of friction between the tires and the road and $0 < x \le 1$. The more slippery the road is, the smaller the value of x. (*Source:* L. Haefner, *Introduction to Transportation Systems.*)

(a) Multiply and simplify the formula for D.

(b) Compare the stopping distance on an icy road with $x = 0.1$ to the stopping distance on dry pavement with $x = 0.4$.

Solution

(a) Because

$$\frac{3600}{30} \cdot \frac{1}{x} = \frac{3600}{30x} = \frac{120 \cdot 30}{x \cdot 30} = \frac{120}{x},$$

it follows that $D = \frac{120}{x}$.

(b) When $x = 0.1$, $D = \frac{120}{0.1} = 1200$ feet, and when $x = 0.4$, $D = \frac{120}{0.4} = 300$ feet. A car traveling at 60 miles per hour on an icy road requires a stopping distance that is 4 times that of a car traveling at the same speed on dry pavement.

Now Try Exercise 73

Division of Rational Expressions

Dividing rational expressions is similar to dividing fractions.

READING CHECK
- How do we divide rational expressions?

QUOTIENTS OF RATIONAL EXPRESSIONS

To divide two rational expressions, multiply by the reciprocal of the divisor. That is,

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C},$$

where B, C, and D are nonzero.

Division is multiplication by the reciprocal.

EXAMPLE 5 Dividing rational expressions

a) $\frac{4}{x+3} \div \frac{2}{5x}$

b) $y+2 \div \frac{y^2-4}{y^2+25}$

c) $\dfrac{3x-3}{x+4} \div \dfrac{3x^2-3}{x^2+6x+8}$

c) $\dfrac{x+2}{x+1}$

14.2 Putting It All Together

CONCEPT	EXPLANATION	EXAMPLES
Multiplication of Rational Expressions	If A, B, C, and D represent rational expressions where B and D are nonzero, multiply the numerators and multiply the denominators: $\dfrac{A}{B} \cdot \dfrac{C}{D} = \dfrac{AC}{BD}.$ Then simplify the result to lowest terms.	$\dfrac{4x}{x-1} \cdot \dfrac{x-1}{x+1} = \dfrac{4x(x-1)}{(x-1)(x+1)} = \dfrac{4x}{x+1}$
Division of Rational Expressions	If A, B, C, and D represent rational expressions where B, C, and D are nonzero, multiply the first expression by the reciprocal of the second expression: $\dfrac{A}{B} \div \dfrac{C}{D} = \dfrac{A}{B} \cdot \dfrac{D}{C}.$ Then simplify the result to lowest terms.	$\dfrac{x+1}{x-3} \div \dfrac{x+1}{x-5} = \dfrac{x+1}{x-3} \cdot \dfrac{x-5}{x+1} = \dfrac{(x+1)(x-5)}{(x-3)(x+1)} = \dfrac{x-5}{x-3}$

14.2 Exercises

CONCEPTS AND VOCABULARY

1. To multiply rational expressions, multiply the _____ and multiply the _____. numerators; denominators

2. To divide two rational expressions, multiply the first expression by the _____ of the second expression. reciprocal

3. $\dfrac{A}{B}\cdot\dfrac{C}{D} = \underline{\dfrac{AC}{BD}}$

4. $\dfrac{A}{B}\div\dfrac{C}{D} = \underline{\dfrac{AD}{BC}}$

REVIEW OF FRACTIONS

Exercises 5–20: Multiply or divide as indicated. Simplify to lowest terms.

5. $\frac{1}{2}\cdot\frac{4}{5}$ $\frac{2}{5}$ **6.** $\frac{6}{7}\cdot\frac{7}{18}$ $\frac{1}{3}$ **7.** $\frac{3}{7}\cdot 4$ $\frac{12}{7}$

8. $5\cdot\frac{4}{5}$ 4 **9.** $\frac{5}{4}\cdot\frac{8}{15}$ $\frac{2}{3}$ **10.** $\frac{3}{10}\cdot\frac{5}{9}$ $\frac{1}{6}$

11. $\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{9}{11}$ $\frac{2}{11}$ **12.** $\frac{2}{5}\cdot\frac{10}{11}\cdot\frac{1}{4}$ $\frac{1}{11}$ **13.** $\frac{2}{3}\div\frac{1}{6}$ 4

14. $\frac{5}{7}\div\frac{5}{8}$ $\frac{8}{7}$ **15.** $\frac{8}{9}\div\frac{5}{3}$ $\frac{8}{15}$ **16.** $\frac{7}{3}\div 6$ $\frac{7}{18}$

17. $8\div\frac{4}{5}$ 10 **18.** $\frac{1}{2}\div\frac{5}{4}\div\frac{2}{5}$ 1

19. $\frac{4}{5}\div\frac{2}{3}\div\frac{1}{2}$ $\frac{12}{5}$ **20.** $\frac{7}{20}\div\frac{14}{5}$ $\frac{1}{8}$

MULTIPLYING RATIONAL EXPRESSIONS

Exercises 21–28: Simplify the expression.

21. $\dfrac{x+5}{x+5}$ 1 **22.** $\dfrac{2x-3}{2x-3}$ 1

23. $\dfrac{(z+1)(z+2)}{(z+4)(z+2)}$ $\frac{z+1}{z+4}$ **24.** $\dfrac{(2z-7)(z+5)}{(3z+5)(z+5)}$ $\frac{2z-7}{3z+5}$

25. $\dfrac{8y(y+7)}{12y(y+7)}$ $\frac{2}{3}$ **26.** $\dfrac{6(y+1)}{12(y+1)}$ $\frac{1}{2}$

27. $\dfrac{x(x+2)(x+3)}{x(x-2)(x+3)}$ $\frac{x+2}{x-2}$ **28.** $\dfrac{2(x+1)(x-1)}{4(x+1)(x-1)}$ $\frac{1}{2}$

Exercises 29–48: Multiply and simplify to lowest terms. Leave your answers in factored form.

29. $\dfrac{8}{x}\cdot\dfrac{x+1}{x}$ $\frac{8(x+1)}{x^2}$ **30.** $\dfrac{7}{2x}\cdot\dfrac{x}{x-1}$ $\frac{7}{2(x-1)}$

31. $\dfrac{8+x}{x}\cdot\dfrac{x-3}{x+8}$ $\frac{x-3}{x}$ **32.** $\dfrac{5x^2+x}{2x-1}\cdot\dfrac{1}{x}$ $\frac{5x+1}{2x-1}$

33. $\dfrac{z+3}{z+4}\cdot\dfrac{z+4}{z-7}$ $\frac{z+3}{z-7}$ **34.** $\dfrac{2z+1}{3z}\cdot\dfrac{3z}{z+2}$ $\frac{2z+1}{z+2}$

35. $\dfrac{5x+1}{3x+2}\cdot\dfrac{3x+2}{5x+1}$ 1 **36.** $\dfrac{x+1}{x+3}\cdot\dfrac{x+3}{x+1}$ 1

37. $\dfrac{(t+1)^2}{t+2}\cdot\dfrac{(t+2)^2}{t+1}$ $(t+1)(t+2)$ **38.** $\dfrac{(t-1)^2}{(t+5)^2}\cdot\dfrac{t+5}{t-1}$ $\frac{t-1}{t+5}$

39. $\dfrac{x^2}{x^2+4}\cdot\dfrac{x+4}{x}$ $\frac{x(x+4)}{x^2+4}$ **40.** $\dfrac{x-1}{x^2}\cdot\dfrac{x^2}{x^2+1}$ $\frac{x-1}{x^2+1}$

41. $\dfrac{z^2-1}{z^2-4}\cdot\dfrac{z-2}{z+1}$ $\frac{z-1}{z+2}$ **42.** $\dfrac{z^2-9}{z-5}\cdot\dfrac{z-5}{z+3}$ $z-3$

43. $\dfrac{y^2-2y}{y^2-1}\cdot\dfrac{y+1}{y-2}$ $\frac{y}{y-1}$ **44.** $\dfrac{y^2-4y}{y+1}\cdot\dfrac{y+1}{y-4}$ y

45. $\dfrac{2x^2-x-3}{3x^2-8x-3}\cdot\dfrac{3x+1}{2x-3}$ $\frac{x+1}{x-3}$

46. $\dfrac{6x^2+11x-2}{3x^2+11x-4}\cdot\dfrac{3x-1}{6x-1}$ $\frac{x+2}{x+4}$

47. $\dfrac{(x-3)^3}{x^2-2x+1}\cdot\dfrac{x-1}{(x-3)^2}$ $\frac{x-3}{x-1}$

48. $\dfrac{x^2+4x+4}{x^2-2x+1}\cdot\dfrac{(x-1)^2}{(x+2)^2}$ 1

DIVIDING RATIONAL EXPRESSIONS

Exercises 49–70: Divide and simplify to lowest terms. Leave your answers in factored form.

49. $\dfrac{2}{x}\div\dfrac{2x+3}{x}$ $\frac{2}{2x+3}$ **50.** $\dfrac{6}{2x}\div\dfrac{x+2}{2x}$ $\frac{6}{x+2}$

51. $\dfrac{x-2}{3x}\div\dfrac{2-x}{6x}$ -2 **52.** $\dfrac{x+1}{2x-1}\div\dfrac{x+1}{x}$ $\frac{x}{2x-1}$

53. $\dfrac{z+2}{z+1}\div\dfrac{z+2}{z-1}$ $\frac{z-1}{z+1}$ **54.** $\dfrac{z+7}{z-4}\div\dfrac{z+7}{z-4}$ 1

55. $\dfrac{3y+4}{2y+1}\div\dfrac{3y+4}{y+2}$ $\frac{y+2}{2y+1}$ **56.** $\dfrac{y+5}{y-2}\div\dfrac{y}{y+3}$ $\frac{(y+5)(y+3)}{y(y-2)}$

57. $\dfrac{t^2-1}{t^2+1}\div\dfrac{t+1}{4}$ $\frac{4(t-1)}{t^2+1}$ **58.** $\dfrac{4}{2t^3}\div\dfrac{8}{t^2}$ $\frac{1}{4t}$

59. $\dfrac{y^2-9}{y^2-25}\div\dfrac{y+3}{y+5}$ $\frac{y-3}{y-5}$ **60.** $\dfrac{y+1}{y-4}\div\dfrac{y^2-1}{y^2-16}$ $\frac{y+4}{y-1}$

61. $\dfrac{2x^2-4x}{2x-1}\div\dfrac{x-2}{2x-1}$ $2x$ **62.** $\dfrac{x-4}{x^2+x}\div\dfrac{5}{x+1}$ $\frac{x-4}{5x}$

63. $\dfrac{2z^2-5z-3}{z^2+z-20}\div\dfrac{z-3}{z-4}$ $\frac{2z+1}{z+5}$

64. $\dfrac{z^2+12z+27}{z^2-5z-14}\div\dfrac{z+3}{z+2}$ $\frac{z+9}{z-7}$

65. $\dfrac{t^2 - 1}{t^2 + 5t - 6} \div (t + 1)$ $\frac{1}{t + 6}$

66. $\dfrac{t^2 - 2t - 3}{t^2 - 5t - 6} \div (t - 3)$ $\frac{1}{t - 6}$

67. $\dfrac{a - b}{a + b} \div \dfrac{a - b}{2a + 3b}$ $\frac{2a + 3b}{a + b}$

68. $\dfrac{x^3 - y^3}{x^2 - y^2} \div \dfrac{x^2 + xy + y^2}{x - y}$ $\frac{x - y}{x + y}$

69. $\dfrac{x - y}{x^2 + 2xy + y^2} \div \dfrac{1}{(x + y)^2}$ $x - y$

70. $\dfrac{a^2 - b^2}{4a^2 - 9b^2} \div \dfrac{a - b}{2a + 3b}$ $\frac{a + b}{2a - 3b}$

71. **Thinking Generally** Simplify $\frac{a - b}{b - c} \cdot \frac{c - b}{b - a}$. 1

72. **Thinking Generally** Simplify $\frac{a - b}{b - c} \div \frac{b - a}{a - b}$. $\frac{a - b}{c - b}$

APPLICATIONS

73. ***Stopping on Slippery Roads*** (Refer to Example 4.) If a car is traveling at 30 miles per hour on a slippery road, then its stopping distance D in feet can be calculated by

$$D = \frac{900}{30} \cdot \frac{1}{x},$$

where x is the coefficient of friction between the tires and the road and $0 < x \leq 1$. (*Source:* L. Haefner.)

(a) Multiply and simplify the formula for D. $D = \frac{30}{x}$

(b) Compare the stopping distance on an icy road with $x = 0.1$ and on dry pavement with $x = 0.4$. 300 ft; 75 ft; dry pavement is one-fourth as long.

74. ***Stopping on Hills*** If a car is traveling at 50 miles per hour on a hill with wet pavement, then its stopping distance D is given by

$$D = \frac{2500}{30} \cdot \frac{1}{x + 0.3},$$

where x equals the slope of the hill. (*Source:* L. Haefner.)

(a) Multiply and simplify the formula for D. $D = \frac{250}{3(x + 0.3)}$

(b) Compare the stopping distance for an uphill slope of $x = 0.1$ to a downhill slope of $x = -0.1$. About 208.3 ft; about 416.7 ft; downhill is twice as long.

75. ***Probability*** Suppose that one jar holds n balls and that a second jar holds $n + 1$ balls. Each jar contains one winning ball.

(a) The probability, or chance, of drawing the winning ball from the first jar and *not* drawing it from the second jar is $\frac{1}{n + 1}$

$$\frac{1}{n} \cdot \frac{n}{n + 1}.$$

Simplify this expression.

(b) Find this probability for $n = 99$. $\frac{1}{100}$

76. ***U.S. AIDS Cases*** The cumulative number of AIDS cases C in the United States from 1982 to 1994 can be modeled by $C = 3200x^2 + 1586$, and the cumulative number of AIDS deaths D from 1982 to 1994 can be modeled by $D = 1900x^2 + 619$. In these equations $x = 0$ corresponds to 1982, $x = 1$ corresponds to 1983, and so on until $x = 12$ corresponds to 1994.* (*Source:* U.S. Department of Health.)

(a) Write the rational expression $\frac{D}{C}$ in terms of x.

(b) Evaluate your expression for $x = 4, 7,$ and 10. Round your answers to the nearest thousandth. Interpret the results.

(c) Explain what the rational expression $\frac{D}{C}$ represents.

WRITING ABOUT MATHEMATICS

77. Explain how to multiply two rational expressions.

78. Explain how to divide two rational expressions.

SECTIONS 14.1 and 14.2

Checking Basic Concepts

1. If possible, evaluate the expression $\frac{3}{x^2 - 1}$ for $x = -1$ and $x = 3$. Undefined; $\frac{3}{8}$

2. Simplify to lowest terms.

(a) $\dfrac{6x^3y^2}{15x^2y^3}$ $\frac{2x}{5y}$ **(b)** $\dfrac{5x - 15}{x - 3}$ 5 **(c)** $\dfrac{x^2 - x - 6}{x^2 + x - 12}$ $\frac{x + 2}{x + 4}$

3. Multiply and simplify to lowest terms.

(a) $\dfrac{4}{3x} \cdot \dfrac{2x}{6}$ $\frac{4}{9}$ **(b)** $\dfrac{2x + 4}{x^2 - 1} \cdot \dfrac{x + 1}{x + 2}$ $\frac{2}{x - 1}$

4. Divide and simplify to lowest terms.

(a) $\dfrac{7}{3z^2} \div \dfrac{14}{5z^3}$ $\frac{5z}{6}$ **(b)** $\dfrac{x^2 + x}{x - 3} \div \dfrac{x}{x - 3}$ $x + 1$

5. ***Waiting in Line*** Customers are waiting in line at a department store. They arrive randomly at an average rate of x per minute. If the clerk can wait on 2 customers per minute, then the average time in minutes spent waiting in line is given by $T = \frac{1}{2 - x}$ for $x < 2$. (*Source:* N. Garber, *Traffic and Highway Engineering.*)

(a) Complete the table.

x	0.5	1.0	1.5	1.9
T	$\frac{2}{3}$	1	2	10

(b) What happens to the waiting time as x increases but remains less than 2? As x nears 2 customers/min, a small increase in x increases the wait dramatically.